UNIT- I

Introduction to Coding Theory

Maharaja Agrasen University, Baddi (HP)

Subject: Coding Theory

Subject Code: MGE(M) 202B

M.Sc. Mathematics (II^{nd} Sem.)

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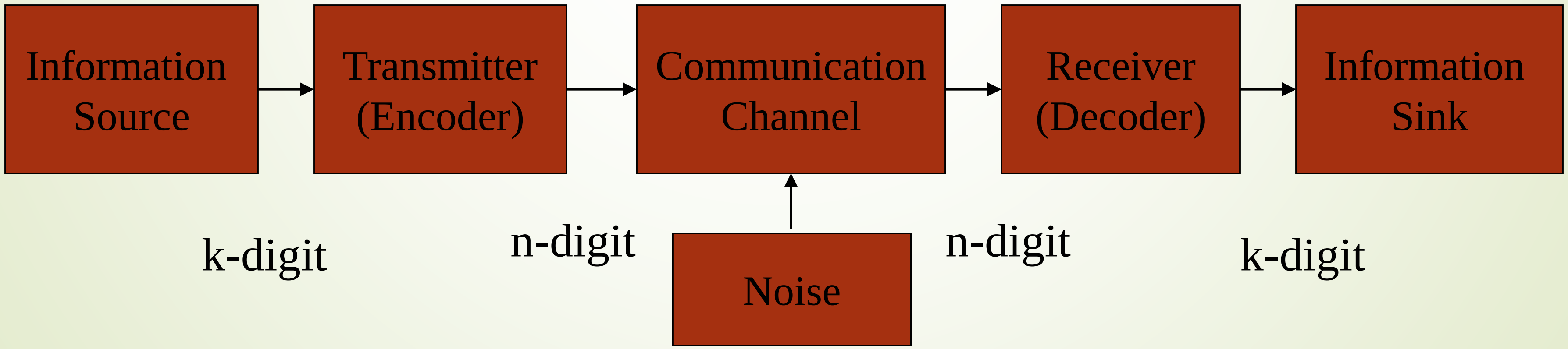
Introduction to Coding Theory

- [1] Introduction
  - Coding theory
    - The study of methods for efficient and accurate transfer of information
    - Detecting and correcting transmission errors
  - Information transmission system



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[2] Basic assumptions

Definitions

- Digit：0 or 1(binary digit)
- Word：a sequence of digits
  - Example：0110101
- Binary code：a set of words
  - Example：1. {00,01,10,11} , 2. {0,01,001}
- Block code ：a code having all its words of the same length
  - Example： {00,01,10,11}, 2 is its length
- Codewords ：words belonging to a given code
- |C| ： Size of a code C(#codewords in C)

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- Assumptions about channel

1. Receiving word by word

2. Identifying the beginning of 1st word

3. The probability of any digit being affected in transmission is the same as the other one.

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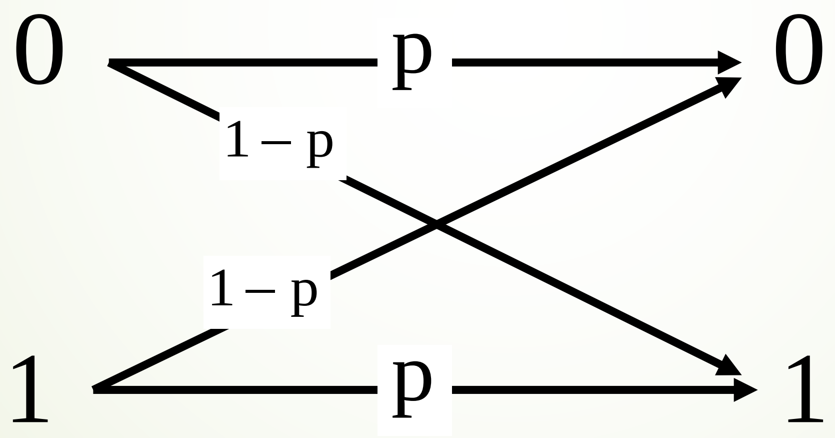
Binary symmetric channel

p: reliability



In many books, p denotes crossover probability.
Here crossover probability(error prob.) is 1-p

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- [3] Correcting and detecting error patterns

  Any received word should be corrected to a codeword that requires as few changes as possible.

$$C_1 = \{00, 01, 10, 11\}$$ Cannot detect any errors !!!

$$C_2 = \{000000, 010101, 101010, 111111\}$$

source → Channel → 110101 —correct→ 010101

$$C_3 = \{000, 011, 101, 110\}$$

parity-check digit

source → Channel → 010 —correct→ 110 ? 000 ? 011 ?

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2. parity-check digit added(Code length becomes 12)

Any single error can be detected !

(3, 5, 7, ..errors can be detected too !)

Pr(at least 2 errors in a word)=$1-p^{12}-12 \times p^{11}(1-p) \fallingdotseq 66 \times 10^{-16}$

So $66 \times 10^{-16} \times 10^{7}/12 \fallingdotseq 5.5 \times 10^{-9}$ wrong words/sec

one word error every 2000 days!

The cost we pay is to reduce a little information rate + retransmission(after error detection!)

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- Example

$v = ? \longrightarrow$ channel $\longrightarrow \varpi = 00110$ $\quad p = 0.98$

v	d (number of disagreements with ϖ)
01101	3
01001	4
10100	2 $\leftarrow$ smallest d
10101	3

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- [7] Some basic algebra

$K = \{0,1\}$

Addition : $0+0=0,\ 0+1=1,\ 1+0=1,\ 1+1=0$

Multiplication : $0\cdot 0=0,\ 1\cdot 0=0,\ 0\cdot 1=0,\ 1\cdot 1=1$

K^n : the set of all binary words of length n

Addition : $01101 + 11001 = 10100$

Scalar multiplication : $0\cdot\omega = 0^n,\ 1\cdot\omega = \omega$

0^n : zero word

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- K^n is a vector space

u, v, w : words of length n

a, b : scalar

1. $v + w \in K^n$
2. $(u + v) + w = u + (v + w)$
3. $v + 0 = 0 + v = v$
4. $v + v' = v' + v = 0, v' \in K^n$
5. $v + w = w + v$
6. $av \in K^n$
7. $a(v + w) = av + aw$
8. $(a + b)v = av + bv$
9. $(ab)v = a(bv)$
10. $1v = v$

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- [8] Weight and distance
  - Hamming weight： $wt(v)$
    - the number of times the digit 1 occurs in v
    - Example： $wt(110101)=4, \quad wt(000000)=0$
  - Hamming distance： $d(v,w)$
    - the number of positions in which v and w disagree
    - Example： $d(01011,00111)=2, \quad d(10110,10110)=0$

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- Some facts：

  u, v, w ： words of length n

  a ： digit

  1. $0 \le wt(v) \le n$
  2. $wt(v) = 0 \quad iff \quad v = 0s$
  3. $0 \le d(v, w) \le n$
  4. $d(v, w) = 0 \quad iff \quad v = w$
  5. $d(v, w) = d(w, v)$
  6. $wt(v + w) \le wt(v) + wt(w)$
  7. $d(v, w) \le d(v, u) + d(u, w)$
  8. $wt(av) = a \cdot wt(v)$
  9. $d(av, aw) = a \cdot d(v, w)$